DOCUMENT RESUME

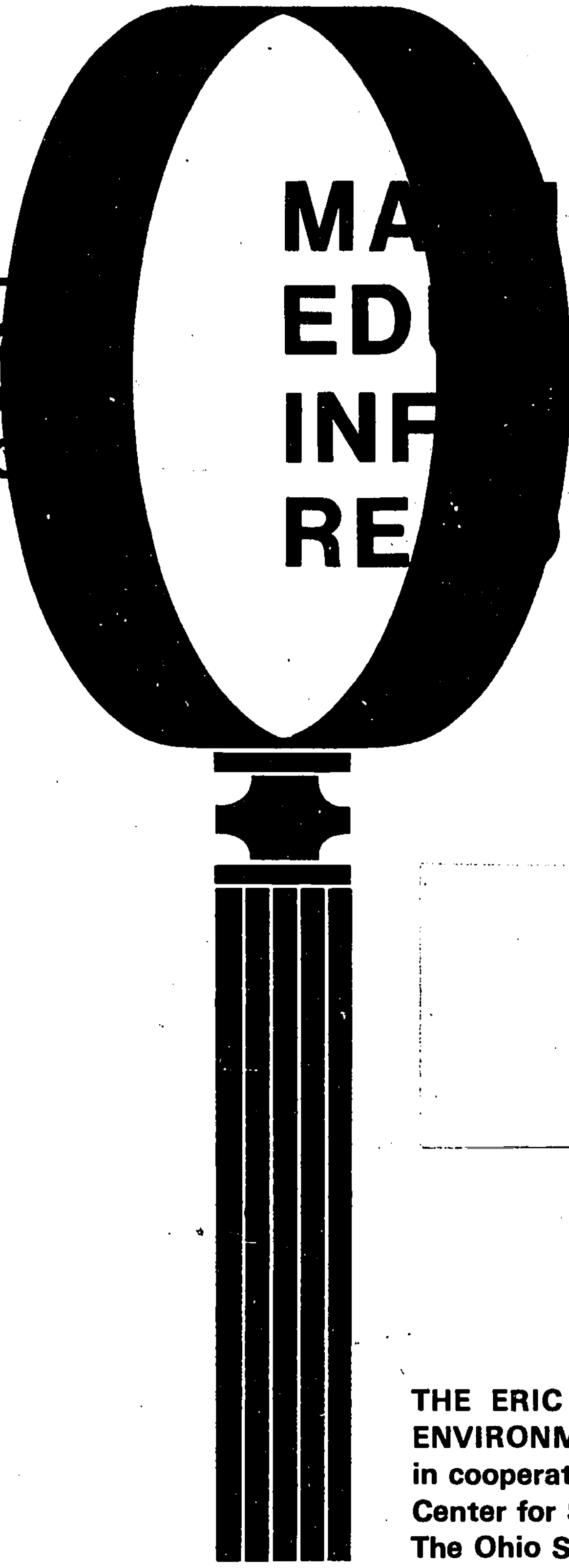
ED 130 861 SE 021 551

AUTHOR Smith, B. Othanel
TITLE Teacher Education in Mathematics.
INSTITUTION ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, Columbus, Ohio.
SPONS AGENCY National Inst. of Education (DHEW), Washington, D.C.
PUB DATE Sep 76
NOTE 24p.; Contains occasional light type
AVAILABLE FROM Information Reference Center (ERIC/IRC), The Ohio State University, 1200 Chambers Road, 3rd Floor, Columbus, Ohio 43212 ($1.25)

EDRS PRICE MF-$0.83 HC-$1.67 Plus Postage.
DESCRIPTORS *Curriculum; *Effective Teaching; Higher Education; *Mathematics Education; *Preservice Education; *Teacher Education

ABSTRACT

An overview of some of the strengths and weaknesses of teacher education programs is presented in this document. Components of teacher education programs are discussed, research in teacher effectiveness is briefly described, and suggestions are given for what is possible and desirable as a program of preparation for mathematics teachers. (DT)



MATHEMATICS EDUCATION INFORMATION REPORT

THE ERIC SCIENCE, MATHEMATICS AND ENVIRONMENTAL EDUCATION CLEARINGHOUSE
in cooperation with
Center for Science and Mathematics Education
The Ohio State University

MATHEMATICS EDUCATION INFORMATION REPORTS

TEACHER EDUCATION IN MATHEMATICS

B. Othanel Smith

ERIC Center for Science, Mathematics,
and Environmental Education
The Ohio State University
1200 Chambers Road, 3rd Floor
Columbus, Ohio 43212

September 1976

In 1976, the Special Interest Group for Research in Mathematics ucation, affiliated with the American Educational Research Association, sponsored a presentation at the annual meeting of the National Council of Teachers of Mathematics. This publication is based on the presentation made by Professor B. Othanel Smith in April 1976 at the NCTM meeting in Atlanta, Georgia.

Professor Smith draws on his years of experience with teacher education programs, and his extensive research work with teachers, to present an overview of some of the strengths and weaknesses of such programs. He concisely presents his position on the need for careful thought about the components of teacher education programs, with some specific suggestions that should be considered by all involved in work with teachers.

ERIC/SMEAC is pleased to make this publication available.

Marilyn N. Suydam
Editor

This publication was prepared pursuant to a contract with the National Institute of Education, U.S. Department of Health, Education and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their judgment in professional and technical matters. Points of view or opinions do not, therefore, necessarily represent official National Institute of Education position or policy.

Mathematics Education Reports

The Mathematics Education Reports series makes available a variety of documents pertaining to research and development efforts in mathematics education. We are pleased to present as part of the series this presentation made at a 1976 conference.

Other Mathematics Education Reports provide information concerning mathematics education documents analyzed at the ERIC Information Analysis Center for Science, Mathematics, and Environmental Education. These reports fall into three broad categories. Research reviews summarize and analyze recent research in specific areas of mathematics education. Resource guides identify and analyze materials and references for use by mathematics teachers at all levels. Special bibliographies announce the availability of documents and review the literature in selected interest areas of mathematics education. Reports in each of these categories may also be targeted for specific subpopulations of the mathematics education community.

Priorities for the development of future Mathematics Education Reports are established by the advisory board of the Center, in cooperation with the National Council of Teachers of Mathematics, the Special Interest Group for Research in Mathematics Education, and other professional groups in mathematics education. Individual comments on past Reports and suggestions for future Reports are always welcomed by the ERIC/SMEAC Center.

TEACHER EDUCATION IN MATHEMATICS

B. Othanel Smith

Many years ago William Hawley Smith described a number of behavioral anomalies in a little book entitled <u>All the Children of All the People</u>. Among the anomalies was a judge who could not tell the time of day and a youngster who could do mathematics like a computer. This was a fascinating book, and in recent years it has reminded me time and again of the fact that individuals do not all learn in the same way, that some need little teacher assistance and others a great deal. Among those who need considerable help are both the bright and the dull, those who come from backgrounds that support school learning as well as backgrounds that negate the school's effect.

THE FOCUS OF TEACHER EDUCATION

I mention these things to underscore the question: What should be the primary aim of teacher education? This question has seldom been considered. If we examine the programs of almost any college of education, we will find that there is little differentiation of preparation in terms of the way students learn, the kinds of learning problems they encounter, or the levels of aspiration they bring to the classroom. Except for special education, differentiation of the teacher education curriculum reflects subject matter divisions and the functions to be performed by the school personnel.

If we look again at the teacher education curriculum, I believe we will see that almost without exception it is geared to the preparation of teachers to teach students who need them least. It can be said with considerable assurance that at least 50 percent of all students can and will

learn what is expected of them if they are provided with learning materials, learning tasks, evaluation of their work, and encouragement.

Almost anyone can be successful, even without professional training, as teachers of these students. This is the strength of the liberal arts argument that the professional preparation of the teacher is of little or no value. As long as we try to make the case for teacher education by reference to the achievement of students who will learn fairly well on their own, we are vulnerable. The test of professionally trained teachers is found in their ability to deal successfully with students who cannot learn without them. If medical doctors were trained to treat only cases that would get well anyway, there would be little point in medical education.

I estimate that a third of the students require the direction of a teacher if they are to acquire the learnings expected of them. Perhaps another 15 percent will need some help, and, as I noted, some 50 percent will do well with very little attention. In my view, therefore, the focus of teacher education should be primarily to prepare teachers to deal with the difficulties and problems that students have as they try to learn. This does not mean that we abandon the general preparation of a teacher for success with students who learn with little aid. But I would curtail the amount of time and energy that now goes into the preparation of teachers to work with these students.

The findings of the first National Assessment of Educational Progress in mathematics (NAEP, 1975) point up my contention. The fundamental processes of arithmetic appear to be mastered by two-thirds to almost all of the 17-year-olds. In simple addition, some 97 percent of the 17-year-olds have mastery, but as the computation becomes more extended the percentage

drops slightly. Yet when the students are asked to add common fractions, say 1/2 plus 1/3, about 34 percent of the 17-year-olds are unable to do the exercise. On tests that require the 17-year-olds to use their knowledge of arithmetic in solving consumer problems, the proportion of those who are successful drops appreciably in comparison to performance on straight computational tasks.

In this paper I am not concerned with this discrepancy, except to point out that mastery of the processes of arithmetic is to be distinguished from the utilization of the knowledge. The ability to use knowledge is a curriculum problem rather than an instructional one. There are persons with Masters' Degrees in Electrical Engineering who do not know how to wire their own homes, and there are many of us who are unable to balance our bank accounts. We learned a long time ago that the mastery of knowledge and the utilization of that knowledge in a different context are two quite different things. So, let us confine ourselves in this discussion to the mastery of mathematical knowledge and operations per se.

We come back then to the question of why teachers are unsuccessful with a tenth to a third of their students in the fundamental processes of arithmetic. Some authorities think that the unsatisfactory achievement is due to an overemphasis upon what is called the "new math." It might well be that there has been a decline in the amount of practice on the fundamental processes, but I do not think that the deficiencies revealed by the National Assessment can be attributed to the new math. The fact is that the adult population performs about the same on the National Assessment exercises as do the 17-year-olds. It is reasonable to assume that the adults were not subjected to the new math, at least not in any large

proportions. Inability in the fundamental processes of arithmetic is of long standing and we have to look elsewhere for the trouble.

WHAT TEACHERS NEED TO KNOW

The view that is taken here is that teachers are professionally unprepared to identify and handle the difficulties of students, and that this accounts for the findings of the National Assessment. What would preparation to deal successfully with the problems of students entail? The first thing that must be noted is the kind of problems that the learner faces. Generally speaking, they are of two kinds: difficult learnings and learning difficulties. Some elements of mathematics are more difficult to learn than others. We call these "difficult learnings" to distinguish them from "learning difficulties."

Over fifty years ago, Thorndike (1922) pointed out that some number combinations were more difficult than others. Nine plus seven, for example, is more difficult than 2 plus 2. He also discovered that materials of instruction provided more practice in the easiest combinations than in the most difficult ones. His studies led to a revision of arithmetic books. Despite Thorndike's auspicious beginning, the problem of determining difficult learnings from bottom to top of the mathematics curriculum has received scant attention. By hypothesis, a list of difficult learnings would constitute one body of information that all teachers should have at their command.

The second body of knowledge that the professional teacher should possess consists of knowledge about learning difficulties. What are these? The first, and perhaps the one most uniquely related to mathematics, is ignorance of prerequisite knowledge. While this type of difficulty has

been recognized for a long time, its recent formulation is associated with the work of Gagné (1962). His view is that learning is hierarchical, that in order to attain a given objective a student must know a number of items beginning with very simple ones and moving to higher and more complex levels until the objective is attained. If students lack any item of knowledge in the hierarchy, they will be handicapped in learning the next item or items.

According to this view, diagnosis consists in preparing tests to determine the elements of knowledge in which students are deficient. Remediation consists in helping them to repair their deficiency. It has long been known, for example, that in order to do long division one must first know how to multiply and subtract. But Gagné's view is that long division should be analyzed to reveal the total hierarchical structure. This requires detailed analysis of the entire operation.

We may disagree with those who would atomize the hierarchy. Perhaps we would prefer to break the hierarchy into bigger steps. But there can be little doubt that in those areas where learning is hierarchical, as is perhaps the case in most mathematics, Gagné's conception is useful. It means that the teacher should be prepared not only in the content of mathematics itself but also in knowledge about the content. The teacher must know not only the content, but also its logical structure.

But lack of prerequisite knowledge is not the only source of learning difficulties. While we cannot go into all of these additional sources, we can point to some general classes and give illustrations. Students bring various kinds of inference patterns to their study of mathematics. Very often students suffer from a tendency to perseverate. They fall into a rut. They try a particular way of working a problem that does not solve

it. Nevertheless, they try it over again and again even though it yields the wrong answer. Students also tend to think easily in terms of direct proportion. But some students find inverse proportion much harder, and the contrapositive pattern "floors" too many of them. The kinds of inference patterns that students bring to the study of mathematics, the kinds they are required to attain, and the relative difficulty of these patterns, has received too little attention both in research and in teacher preparation.

I once worked with a student who was having trouble in plane geometry. I found that he could not follow a proof as it was being explained to him, let alone develop the formulation of a proof. But when I took the same course of reasoning into a semantic context with which he was familiar, he could follow the steps. Was his difficulty due to a semantic overload in the geometry context? Was it due to inability to do the more formal reasoning of geometry, although he was successful in material reasoning? I am not familiar with much of the research on learning in mathematics, but I hope that this sort of difficulty has been thoroughly studied. Be that as it may, this case, I believe, exemplifies part of what I mean by student difficulties rooted in inference patterns.

Some students are unable to learn the mathematics expected of them because of emotional difficulties. From prior experience they have somehow come to be frightened by mathematics. They perceive it as hard--something they are not capable of doing. If a student perceives himself as incapable, he is likely to live up to his own estimation. The identification of these students by means of self-concept tests designed for mathematics is an important diagnostic step. And the success of these students will be dependent upon the ability of the teacher to improve their self-assessment.

Reading ability is of course an important factor in translation problems. A poor reader will certainly have difficulties when he is given a verbal task. The identification of students who are suffering from reading deficiency is another aspect of diagnosis with which the teacher must be familiar. The question of who is responsible for the removal of the deficiency may be debatable. But the teacher of mathematics cannot escape the responsibility for identifying those students whose inability to do translation problems is rooted in reading difficulties.

There are of course all sorts of physical dysfunctions that the mathematics teacher should be aware of. Among these are visual and auditory deficiencies, mental retardation, and anemia. The mathematics teacher should be able to identify students at least with these dysfunctions, for screening purposes and as mainstreaming becomes more and more prevalent, in order to be able to engage in proper instruction.

There are many other sources of difficulty in learning mathematics, but these examples should be sufficient to indicate what is entailed by a teacher education program that attempts to prepare teachers to work with students who cannot learn without their assistance.

PREPARATION IN CONCEPTS

Let me now consider the question of how the teacher is to be prepared. Teaching training has at least three facets: mathematical knowledge, pedagogical concepts and principles, and teaching skills. I shall not deal with the question of what mathematical knowledge the teacher should possess. Suffice it to say that some teachers are ill-prepared in mathematical concepts and processes.

The importance of pedagogical concepts has already been suggested in my earlier remarks. Among the concepts that a teacher should possess are learning hierarchy, prerequisite knowledge, inference patterns, forms of knowledge, logical operations, self-concept, perseveration, reinforcement, difficult learnings, and a host of others pertaining to reading, classroom management and discipline, conduct of instruction, and testing (Hudgins, 1974).

These elements of teacher training are sometimes referred to as theory and their significance thereby dismissed. The tendency to disregard these elements of the teacher's preparation is due in part to the influence of radical behaviorism and to the belief that what counts most in teacher preparation is experience in the classroom. It is difficult to defend the preparation of teachers in pedagogical concepts by reference to behavior, because the concepts function covertly. While their covert influence is hard to identify in behavior, it is easy to recognize in our own experience. We often start to say something in the classroom--to explain or delineate a point--and, before we utter a word, we decide to say something different. This internal activity is not evident to an observer. As teachers work with students they are constantly modifying their performance in terms of their perception of student behavior and their interpretation of it. These interpretations go back to the kinds of concepts the teacher has; the more technical the concepts, the more appropriate are the teacher's responses likely to be.

If conceptual knowledge is effective in teaching, why is there so much opposition to it? It is often depreciated not only by teachers but also by some members of teacher education faculties. Those who have made

studies of teachers' talk (e.g., Jackson, 1968) tell us that teachers use the language of common parlance rather than the technical language of the profession. It seems reasonable to suppose that if they do not use the language by which the concepts are known, they likely make little or no use of the concepts themselves. This is a distressing conclusion, one that strikes close to the heart of teacher education itself.

This conclusion is probably true and we must face the question of why pedagogical concepts, underwritten by research, are so infrequently used. I believe it is to be attributed to the fact that these concepts are taught verbally and learned at a verbal level. A concept is a basket into which we can put a number of similar items. To tell which items belong in the basket and which do not is to have a set of criteria. These criteria are the rules of exclusion and inclusion, and they must specify observable properties or otherwise the concepts are not useful in a practical context.

Now, it is possible to acquire a concept verbally; that is, to know the name of a class and the rules for deciding what is to be included or excluded from the class. We can know all of this at a verbal level and have not the slightest ability to identify the objects to be included in the class. As one teacher in training put the matter, "I know what the word motivation means. I have read about it in the text. But how can I tell whether my students are motivated or not?" This same question can be asked about almost any pedagogical concept, and the failure to provide an answer to the question is one of the major defects in teacher education.

Recent advances in technology have made it possible to bridge the gap between the verbal learning of pedagogical concepts and the concrete world

of experience to which they pertain (Borg, 1975a, 1975b). This is now done by capturing episodes of teacher and pupil behavior on films. We can now stage a behavioral situation exemplifying a concept, film this situation, and prepare a package of instructional materials to accompany the film. These can enable the student to master the concept in behavioral terms as well as verbally. It is one thing to read about a concept in a text or to hear it explicated by an instructor with a few verbal instances. It is quite a different thing to see that same concept manifested in behavior in the form of its defining attributes. The test of mastery of the concept is not the ability of the student to define it verbally, although this is not to be depreciated, but the test is the student's ability to recognize the behavioral attributes of the concept in other filmed situations and ultimately in the classroom itself.

Some evidence indicates that when concepts are taught in this way, they can influence the teacher's performance (Wright *et al.*, 1970). There is reason to believe also that the language of the teacher will thereby become professional.

WHAT PERFORMANCE MAKES A DIFFERENCE?

Turning now to performance per se, practice is the time-honored way of improving it. There are three questions about practice on which research has had something to say. These questions are: What skills should be developed through practice? Under what conditions is practice most apt to be effective? Does training make any difference anyway? The findings of research on these questions is often conflicting and controversial, but there are a few points that appear to stand up and further research will likely yield others.

The question of what skills teachers should acquire is provisionally answered partly by research and partly by the practical wisdom of the profession. In our present state of knowledge, the skills we require teachers to practice are typically determined by wisdom rather than research. We are all by now familiar with Rosenshine and Furst's (1971) list of variables gleaned from an analysis of process-product studies. The ones that relate most directly to teacher behavior are: clarity, variability, enthusiasm, business-like behavior, use of student ideas, criticism, use of structuring comments, types of questions, and probing. The support these variables enjoy in the research literature varies in strength. For example, the proposition that "teachers who ask more high-order questions than low-order questions are more successful, in terms of student achievement, than teachers who ask a greater proportion of low-order questions" is not sustained by research. Some studies support the proposition and some do not. On the contrary, the research findings support the proposition that "lower proportions of high-order questions result in more student knowledge and comprehension than do higher proportions of questions that provoke thought." Teacher acceptance of student's answers is found to be positively associated with increased student achievement. But clarity, for instance, is not found to be consistently conducive to achievement.

The analysis of research on these variables by Rosenshine and Furst has been criticised by Heath and Nielson (1974) and Dunkin and Biddle (1974). Little more need be said here. Suffice it to note that the variables are vague and perhaps complex (Dunkin and Biddle, 1974). For this reason one must take care in reducing them to practice. For example, in the studies reported by Rosenshine and Furst, clarity is variously defined.

In some studies it pertains to presentation, in others to the easiness of the points made by the teacher, to explanation of concepts and facility with materials, and to the cognitive level of the teacher's discourse. Perhaps all of these are different aspects of clarity, but the problem of teasing out the particular meanings of these for teacher behavior is a given content such as mathematics remains to be done.

CONDITIONS OF EFFECTIVE TRAINING

Under what conditions can we expect practice to lead to improvement? It should be borne in mind that beginners bring to the teaching situation habits of interaction and explanations of behavior inculcated from years of experience, beginning almost at birth. Some of these habits and concepts are in line with effective teaching behavior; others must be modified or eliminated, as new modes of behavior are learned.

It should also be borne in mind that teaching situations, even tutorial ones, are not simple. The teacher must learn to analyze them and to perceive behavior objectively. To look at behavior as being good or bad--that is, to pass judgment upon it--is to preclude the possibility of understanding it. The teacher must learn not only to view the student's behavior objectively, but also to interpret it in terms of technical concepts and principles. These, it seems to me, are part of the necessary conditions for practice to be effective.

Another condition, supported not only by professional wisdom but also by research, is that the teacher receive feedback. Whether the feedback is given immediately following the practice or given later as the teacher views a videotape of his teaching appears to matter little. But without feedback, improvement from practice is not to be expected. The feedback

must be specific and to the particular point of the teacher's behavior. Furthermore, if the teacher does not have enough knowledge to make use of the feedback, little or no improvement will occur.

The technical means by which these requirements are to be satisfied must be worked out if the practice of the beginning teacher, or for that matter, the teacher in service, is to pay off in greater student achievement. I believe that the conventional system of student teaching does not satisfy these conditions. I do not wish to be understood as saying that practice teaching in actual classroom situations is to be discarded. On the contrary, it is a necessary ingredient of teacher preparation, but it should follow upon systematic practice under more controlled conditions.

DOES TRAINING MAKE A DIFFERENCE?

We come now to the final question; namely, does training make a difference? The work of Coleman on inequalities in education shows a very small positive relationship between teacher preparation and student achievement in the tools of learning. Some persons have concluded from this study that the value of teacher-training is questionable if not worthless.

Partly as a result of the uneasiness created by the Coleman report, a number of recent studies have been made to determine the effectiveness of training. A recent study (Clark, 1976) of the effect of practice compares the effects, as measured in terms of pupil achievement, of twelve trained and experienced teachers, over a number of periods of teaching in small-group situations. They were given specified teaching materials and the instructional objectives. The teachers decided the mode of instruction themselves. Three teaching sessions were provided for each class.

The teaching variables were analyzed from records of the teaching process. When these were examined it was found that they were very similar from one day to another, and that there was close similarity between the teaching process variables from teacher to teacher. These teachers apparently learned very little from their experience and they unknowingly taught alike. Furthermore, only a few of the teachers showed marked increases in student achievement. Generally speaking, classes on the first and second days scored higher on recall tests and classes on the third day scored lower. On essay tests the scores were stable across teachers and days. On its face, this study appears to underscore the proposition that learning to teach is a vulnerable notion.

We should be cautious about drawing conclusions from research on the effects of uninformed practice at this stage of the game. The studies are fraught with further questions. They should be viewed as part of a continuing effort to explore teaching behavior--how it can be changed and what variables affect student achievement and their attitude and feelings toward instruction and learning.

I wish, however, to make a few observations. In the first place, we have known for a long time that mere experience bears little positive relationship to improvement of instruction. The problem is not whether uninformed practice makes a difference but how and about what are teachers to be informed and whether informed teachers can make a difference in their practice. Furthermore, as long as our research uses students who will learn what is expected of them by almost any mode of teaching, or even by themselves, we can expect changes in teacher behavior to have little or no effect upon achievement. The real test of teacher training will be found

in studies that use as subjects students who require the assistance of the teacher in order to learn what is expected of them. Such teacher assistance will, I believe, entail diagnostic and remedial measures, many of which have yet to be worked out.

WHAT DOES ALL THIS ADD UP TO?

Fortunately we do not have to stop the education of teachers until the questions that you and I have in mind have been answered by research. It is incumbent on us to improve programs of training that we now have by recourse to as much knowledge as we can gain from research and our practical experience. All that I can do here is to lay before you one person's opinion, my own, of what is now possible and desirable as a program of preparation for mathematics teachers, or for that matter teachers of other content subjects as well. This is what I propose:

1. We should identify, consistently define, and teach to the point of mastery those concepts which enable a teacher to identify and understand the difficulties that students have in learning. This task will entail the identification of all of the learning difficulties that research and wisdom of the profession now make possible, the classification of these difficulties into kinds, so as to enable the teacher to approach teaching-learning situations with a systematized body of pedagogical concepts.

2. These concepts should be taught not only verbally but by means of videotapes and films that exemplify the criterial attributes of the concepts. Packages of instructional materials should be prepared for use by the student in observing and analyzing the videotapes or films until they have mastered the defining attributes. Mastery should be determined by the

student's ability to identify these attributes not only in additional films but also in normal classroom situations.

3. The skills and techniques of diagnosis and remediation should be identified and consistently defined. It will be necessary to comb the research literature as well as residues of practical experience in order to identify these elements of training. While these techniques and skills should be those for which there is reason to believe that they are effective in helping students overcome barriers to their learning, a few should be generic in the sense that they pertain to general modes of instruction.

4. Instructional materials must be contrived that will provide training in these techniques and skills under controlled conditions. These conditions can be instituted by means of videotapes and transcribed classroom discourse. By these means the individual can learn to observe his own teaching objectively, to study it, to gain the advantages of feedback, and to learn what he needs to know in order to take advantage of the feedback in improving his teaching performance.

5. Training in the concepts, techniques, and skills should begin early in the teacher's undergraduate preparation--surely not later than the beginning of the junior year. This training should be conducted in laboratories that provide adequate materials and technical means. At the same time the student should be expected to participate intermittently in the work of the school as an observer and teacher aide.

6. In order to accomplish the foregoing things it will be necessary to eliminate, at the undergraduate level, many general courses such as "introduction to education" so that the time of the student can be concentrated on the task of learning to teach. That after all is our primary

responsibility. This will mean that a great proportion of the faculty of teacher institutions must become teacher trainers and cease being purveyors of information about schools and society, pedagogical ideologies and formulas that from time to time sweep through the schools. All these matters that may be relevant to the work of the teacher are better acquired in the post-baccalaureate years.

7. All that I have been trying to say can be summed up in a general way. Many of us have been critical of teachers for their failure to use the knowledge resulting from research. We have even insisted, in some cases, that they themselves become research-minded as teachers. I am suggesting that we follow our own prescription, that we use the knowledge from research about how and what teachers should be taught in order to improve the teacher training programs that we now have.

REFERENCES

Borg, Walter R. Moving Towards a Breakthrough in Teacher Education. Education 95: 302-323; 1975a.

Borg, Walter R. Protocol Materials as Related to Teacher Performance and Pupil Achievement. Journal of Educational Research 69: 23-30; 1975b.

Clark, Christopher M. The Effects of Teacher Practice on Student Learning and Attitudes in Small Group Instruction. Technical Report No. 47. Stanford, California: Stanford Center for Research and Development in Teaching, 1976.

Dunkin, Michael J. and Biddle, Bruce J. The Study of Teaching. New York: Holt, Rinehart and Winston, 1974. passim.

Gagne, Robert. The Acquisition of Knowledge. Psychological Review 69: 355-365; 1962.

Heath, R. W. and Nielson, M. A. The Research Basis for Performance-Based Teacher Education. Review of Educational Research 44: 463-484; 1974.

Hudgins, Bryce B. A Catalogue of Concepts in the Pedagogical Domain of Teacher Education. Albany, New York: Multi-state Consortium on Performance-Based Teacher Education, New York State Education Department, Division of Teacher Education and Certification, 1974.

Jackson, Phillip W. Life in the Classroom. New York: Holt, Rinehart and Winston, 1968.

National Assessment of Educational Progress. _Math Fundamentals: Selected Results from the First National Assessment of Mathematics._ Denver, Colorado: Education Commission of the States, January 1975.

Rosenshine, Barak and Furst, Norma. Research on Teacher Performance Criteria. In _Research in Teacher Education_. (Edited by B. Othanel Smith). Englewood Cliffs, New Jersey: Prentice-Hall, 1971.

Thorndike, E. L. _The Psychology of Arithmetic_. New York: Macmillan, 1922.

Wright, C. J.; Nuthall, G. A.; and Laurence, P. J. A Study of Classroom Interaction in the Training of Teachers. _Educational Research Newsletter_. Christ Church, New Zealand: Department of Education, University of Canterbury, 1970.